ALGORITHM FOR FINITE ELEMENT ANALYSIS OF FUNCTIONALLY GRADED MATERIALS

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ABSTRACT
Functionally graded materials (FGM) are the unique class of materials whose properties vary within the object due to continuous material variation in the heterogeneous object (HO). Rapid Prototyping processes have potential to fabricate such objects but require a CAD model with supplementary material information. In the current work, a System structure is proposed for CAD modeling of FGM objects. A 'Gradient Reference' based CAD approach is developed for modeling material distributions in the graded regions for explicit applications. Finally, the properties or behavior of FGM objects are analyzed using finite element approach.
Keywords: Heterogeneous Objects, functionally graded material, gradient reference, rapid prototyping, finite element analysis.

1. INTRODUCTION

Traditional CAD systems, used for conventional design method, can only represent the geometry and topology of an object. No material information is available within the representation, required for the fabrication of heterogeneous objects (HO). Rapid prototyping (RP) techniques allow heterogeneous objects to be produced using 3D CAD models by varying material composition region-wise, layer-wise, or point-wise. The required 3D CAD model should have not only the geometric information but also the information of material property, etc. at each point inside an object. With the capability to fabricate heterogeneous objects, more functionally efficient and cost reducing objects called functionally graded material objects can be realized. The term functionally graded material is a distinctive class of materials in which two or more materials with different characteristics are continuously distributed without any specific boundary interface and properties of the materials are controlled according to functional requirement of the object. The primary focus of the recent research is to develop a CAD modeling system for FGM objects which should at least meet the following specifications [8]:

- Intuitive in representing geometry, topology and material information simultaneously.
- Capable of representing complex solids: the solids to be modeled may be complex in geometry as well as in material variations.

- Compact and exact: the representation should be compact, and both the geometry and material information can be retrieved accurately and efficiently.
- The representation of material properties must be compatible with current or proposed standards for geometric modeling representations as described in ISO 10033. This is essential to exchange data among design, analysis and manufacturing process plan domains.

Keeping in view the above credentials, an effective CAD modeling approach for FGM objects is proposed in this work. The primary goal of the present research is to develop systematic CAD modeling methodology and to analysis the FGM object using finite element approach. The paper is organized as follows: in Section 2, the previous work is reviewed; Section 3 describes a system structure for integration of with other modules; Mathematical model for heterogeneous object is formulated in Section 4; Section 5 represents the algorithm for finite element analysis of FGM and the final section is conclusion with future scope to extend this work.

2. REVIEW OF PREVIOUS RESEARCH

Recent studies show that an effective heterogeneous CAD modeling system should at least meet the various specifications [8]. Approaches of modeling of HO have been extensively studied in computer and manufacturing community. Kumar and Dutta [9-10] proposed an approach to model multi-material objects based on R-m sets and R-m classes primarily for application in layered manufacturing. Boolean operators were defined to facilitate the modeling process. Jackson [7] and Liu [11-12] proposed a local composition control (LCC) approach to represent heterogeneous object in which a mesh model is divided into tetrahedrons and different material compositions are evaluated on the nodes of the tetrahedrons by using Bernstein polynomials. Chiu [2] developed material tree structure to store different compositions of an object. The material tree was then added to a data file to construct a modified format being suitable for RP manufacturing. Siu and Tan [16] developed a scheme named 'source-based' method to distribute material primitives, which can vary any material with an object. The feature-based modeling scheme was extended to heterogeneous object representation through boundary conditions of a virtual diffusion problem in the solid, and then designers could use it to control the material distribution and described by Qian and

Dutta [14-15]. Liu extended his work in by taking parameterized functions in terms of distance(s) and functions using Laplace equation to smoothly blend various boundary conditions, through which designers could edit geometry and composition simultaneously [11-12]. Kou and Tan [8] suggested a hierarchical representation for heterogeneous object modeling by using B-rep to represent geometry and a heterogeneous feature tree to express the material distributions. Various methods for designing and optimizing objects composed of multiple regions with continuously varying material properties have been developed. Biswas and Wang [1],[16] proposed a level-set based variational scheme. Tsukenov [17] presented a mesh-free approach based on the generalized Taylor series expansion of a distance field to model and analyze a heterogeneous object satisfying the prescribed material conditions on a finite collection of material features and global constraints. However, almost all of the research interests are mainly focused on the computer representation of heterogeneous object, rather than the procedure for rapid prototyping and fabrication of heterogeneous object. The approaches were verified in commercial software packages, such as Solidworks and Unigraphics [11],[14]. A commercial CAD package independent system is developed to deal with the HO modeling, but not including the slicing procedure for RP manufacturing [13],[15]. CAD gradient reference model with systematic methodologies for representation, visualization and manipulation of heterogeneous objects was developed [3-6]. A detailed description of each module cannot be presented for the paper length.

3. ANALYSIS OF FGM

Finite element analysis (FEA) software packages prove to be powerful tools in design evaluations and can be adapted and applied to evaluate the functions/performances of heterogeneous objects. In this section, the basic procedure for finite element analysis of heterogeneous objects is discussed, as described in fig.1.

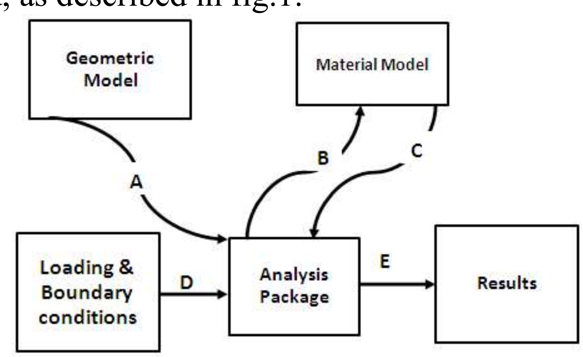

Fig.1: A: Mesh Generation and Refinement, B: Material Evaluation of Each Element and Property Evaluation for Each Element, C: Analysis of Each Element and Assembly of Elements, D: Loading and Boundary Conditions, E: Post processor.

The geometric model of the object is first converted into a neutral file format (e.g. DXF, ACIS or IGES format), and then exported to analysis software (e.g. ANSYS) for finite element mesh generation and refinement based on the object's geometric information (process A). The meshed geometric model is send to material model for evaluating material compositions of the object for each element using scan algorithm and to determine relevant material properties at each location in HO (process B). The resulting model is exported to analysis software where boundary conditions/initial conditions and loadings are applied for analysis of HO (process C and D). The local stiffness matrix for each element is then determined for getting the results. Elements are assembled using global stiffness matrix and finally results for HO are displayed using post processing (process E).

The above descriptions outlined the basic procedures of the data communications between developed integrated model and analysis software. ANSYS support ACIS file format for processing of HO. The material information exchanges are also possible through data files with extra programming works (e.g. programming with Visual C++).

1. MATHEMATICAL MODEL FOR PROPERTY EVALUATION IN FGM

The meshed geometric model is scanned using Z-buffer algorithm to evaluate material composition for each element using mathematical model. The property of FGM unit volume for each element is then determined using Voigt's rule given by Eqn. (1).

$$S = \sum_{j=1}^{m}\sum_{r=1}^{k} V_j S_r \tag{1}$$

Where S is the property of heterogeneous volume fraction
V_j is the volume fraction of each material in unit volume
S_r is the property of r^{th} material.

For two materials composition, the heterogeneous property is defined by Eqn. (2).

$$S = V_1S_1 + (1- V_1)\, S_2 \tag{2}$$

Sharp material changes along the component boundaries results in abrupt property variations. So for the smooth material properties variation, one of the available blending functions may be used. Blending can be used to get a smooth material transition in FGM from one end to other end. It can blend not only the geometric shape but also the property requirements at specific point. In our case, the constant blending functions are incorporated at end positions and in between, a blending function, f_b, along with the distance function is used to avoid the sharp change in material properties, see Eqn. (3).

$$S = f(s)V_1S_1 + (1- V_1)\,(1-f(s))f_b\, S_2 \tag{3}$$

2. ALGORITHM FOR FEA FORMULATION FOR FGM OBJECT

Finite element analysis have been successfully used in analyzing the structural, thermal, electric, kinematic and other responses for designed parts or assemblies. Many existing FEA approaches are focused on homogeneous object analysis. The primary reason is possibly due to few applications of heterogeneous objects in the past. Generally, FEA tools can be adapted and applied to evaluate the functions/performances of heterogeneous objects. Problem of finite element structural analysis of FGM object is formulated in this work. The heterogeneous object is first discretized by selecting type of element. The displacements matrix U for a specific problem for a chosen co-ordinate system is defined as:

$$\mathbf{U = [u_a\ u_b\ u_c] = NQ} \tag{4}$$

where a,b,c are co-ordinates of chosen co-ordinate system, N is shape function matrix and Q nodal displacement vector .

In this, the displacement vector can be defined as:

$$\mathbf{Q=[u_{a1}\ u_{b1}\ u_{c1}\ldots\ldots\ldots,u_{an}\ u_{bn}\ u_{cn}]} \tag{5}$$

where n is number of nodes.

N, the shape function is defined as:

$$N = \begin{bmatrix} N_1 & 0 & 0 \ldots\ldots\ldots & N_n & 0 & 0 \\ 0 & N_1 & 0 \ldots\ldots\ldots & 0 & N_n & 0 \\ 0 & 0 & N_1 \ldots\ldots\ldots & 0 & 0 & N_n \end{bmatrix} \quad (6)$$

Then strain –displacement relations are established as:

$$\boldsymbol{\varepsilon = LU} \quad \textbf{(7)}$$

where ε denotes strain vector and is given by $[\varepsilon_a\ \varepsilon_b\ \varepsilon_c\ \gamma_a\ \gamma_b\ \gamma_c]$, where ε_i and γ_i are normal and shear strains. L is operation matrix and depends on specific problem and choice of co-ordinate system.

Strain – displacement relations can also be established as:

$$\boldsymbol{\varepsilon = BQ} \quad \textbf{(8)}$$

where B is strain – nodal displacement matrix and is defined as:

$$\mathbf{B=LN} \quad \textbf{(9)}$$

Similarly stress-strain relations can be established as:

$$\boldsymbol{\sigma = E_e\ \varepsilon} \quad \textbf{(10)}$$

where E_e is the effective elasticity matrix for FGM which can be calculated using Eqn. 10 and stress matrix, combination of normal stress (σ_i) and shear stress $(\tau)_i$ is defined as $\boldsymbol{\sigma = [\sigma_a\ \sigma_b\ \sigma_c\ \tau_a\ \tau_b\ \tau_c]}$

Finally the effective stiffness matrix for FGM objects is defined as:

$$\mathbf{K_e = \int_a\int_b\int_c B^T E_e B\ da\ db\ dc} \quad \textbf{(11)}$$

3. CONCLUSION AND FUTURE SCOPE

This work presents a gradient reference approach for modeling heterogeneous object with complex geometry and simple material variations. The basic structure listing sequence of CAD data flow to visualize HO on graphical user interface and procedure for processing of HO for rapid prototyping processes are discussed. The distribution of material is obtained by using different material information functions. The proposed gradient reference approach represents intricate geometries as well as material variation simultaneously; ensures smooth material variations throughout the complex object; imposes independent material alterations on the cells so that their original properties can be properly retained in the resultant object; offers local control on material distribution; consistent in data representation; and computationally robust and efficient. Finally finite element approach is used to analyze the properties of FGM object.

The present work can be extended to address multi-material distribution issues for the fabrication of HO using rapid prototyping techniques. Further the work can be extended and implemented to complex and irregular material distributions. The approach can be used for object modeling i.e. solid modeling with other physical attributes such as mechanical properties, material distribution etc. Dynamic heterogeneous objects (DHO) are the new class of heterogeneous objects. Unlike current heterogeneous object modeling, DHO deals with space dependent heterogeneities and time dependent shapes and material distributions. By taking time into consideration, more realistic process simulation can be achieved. DHO technology has emerging applications in life science domain, biomedical applications, dynamic process simulation and bio-CAD etc.

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